Investigations of β^--Decay Half-life and Delayed Neutron Emission with uncertainty analysis[*]

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β-decay half-life and the β-delayed neutron emission (βn) are of great importance in the development of basic science and industrial applications, such as nuclear physics and nuclear energy, where β^--decay plays an important role. Many theoretical models have been proposed to describe β-decay half-life, while the systematic study on βn is still rare. This paper aims to investigate β^--decay half-lives and βn probabilities through analytical formulas and compare them with the experimental data. Analytical formulas of the β^--decay properties have been proposed in considering the prominent factors: decay energy, odevity, and shell effect. The bootstrap method is used to simultaneously evaluate the total uncertainty of calculations, composed of the statistic and systematic uncertainties. The β^--decay half-lives, βn probabilities, and the corresponding uncertainties have been evaluated for the neutron-rich region. The experimental half-lives are well reproduced. More predictions are also presented with theoretical uncertainties, which helps better understand the disparity between experimental and theoretical results.

Keywords: Neutron-rich nucleus, β-delayed neutron emissions, Bootstrap method

I. INTRODUCTION

β-decay of exotic nuclei, especially approaching the limits of the nuclear landscape, has been focused on in recent decades [1–10]. It is still a complex problem to estimate related properties with satisfactory accuracy owing to the complexity of nuclear structure and interactions between nucleons [11].

The β^--decay half-life and β^--delayed neutron emission (βn) probability are considered to be important input parameters in nuclear reactors, nuclear structure, as well as astrophysical scenarios [12, 13]. Based on the parameterization of observed properties of atomic nuclei, many approaches have been successively proposed for β-decay [14].

Some methods made use of quasiparticle random phase approximation (QRPA) with the semi-empirical models [15–17] while some others have proceeded from a microscopic perspective, such as the Hartree-Fock-Bogoliubov framework [18–21]. Those microscopic calculations, even with certain approximations on interactions and/or wavefunctions, are still time-consuming. Neat and reliable formulas are beneficial for the β-decay study. Phenomenological models have been widely proposed [14, 22–25].

Along with the development of high-precision experimental techniques, we are gradually making up for the deficiency in experimental data of β^--decay, especially for nuclei near the neutron shell closures of 50 and 82 [26–29]. Therefore, a new systematic study is of interest and helps to understand the β-delayed particle emission more deeply with reliable calculations.

This work focuses on the β^--decay half-life $T_{1/2}$ and βn probability, $P_{\beta n}$, for neutron-rich nuclei. Analytical formulas of the β^--decay strength are proposed based on the standard β-decay theory with further physics considerations on decay energy, odevity, and shell effect. Moreover, the bootstrap method is used to evaluate the uncertainties of calculations.

According to previous studies of β^--decay, the contribution of the first-forbidden (FF) transition to the total decay rate has been found not negligible for nuclei far from the β-stable valley [17, 30, 31], for which we equate FF branches and other forbidden transitions to one or two effective Gamow-Teller (GT) branches in this study. For a better study of β^--delayed neutrons, this work only concerns neutron-rich nuclei with Z = 29∼57, which includes important fission products and the precursors of the delayed neutron in the nuclear reactor [32]. The experimental data used for this work come from the newest compilation and evaluation of β-delayed neutron emission probabilities and half-lives for Z > 28 precursors provided by the AME-2020 [33, 34].

The main purposes of this work are: to determine parameters of the analytical formulas to describe $T_{1/2}$ and $P_{\beta n}$; to estimate the uncertainties of corresponding formulas with the bootstrap method; and to make predictions for the nuclei without experimental data of $T_{1/2}$ and $P_{\beta n}$.

II. METHODS

A. Theoretical derivation and formulas

This work focuses on the study of the half-lives of the β^--decay and the βn probabilities of neutron-rich nuclei. According to Fermi's β-decay theory, the standard formula of the β^--decay half-life $T_{1/2}$ through Fermi transition and GT transition is as follows [35]:

$$T_{1/2} = \frac{\kappa}{f_0(B_{\mathrm{F}} + B_{\mathrm{GT}})}, \tag{1}$$

where $\kappa \equiv \frac{2\pi^3\hbar^7 \ln 2}{m_e^5 c^4 G_F^2} = 6147$ s, B_{F} and B_{GT} are respectively the reduced transition probability term for Fermi transition and GT transition. f_0 is the phase-space factor, also called Fermi integral, which is expressed for β^- decay:

$$f_0 = \int_1^{E_0} F_0(Z_f, \epsilon) p\epsilon (E_0 - \epsilon)^2 d\epsilon, \tag{2}$$

where $\epsilon \equiv \frac{E_e}{m_e c^2}$, $E_0 \equiv \frac{E_i - E_f}{m_e c^2}$, $p \equiv \sqrt{\epsilon^2 - 1}$. E_e is the total energy of released electrons during the decay, Z_f is the number of protons of the daughter nucleus. Then, E_i and E_f are the energy levels of the initial and final states whose difference $(E_i - E_f)$ is consequently the decay energy Q_β and F_0 is the Fermi function.

The Primakoff-Rosen's approximation [36] is applied to Eq. (2), simplifying Eq. (1) to a manipulable form, which also imposes a relatively high value of decay energy of β-decay for the constraint on the selection of experimental data. All nuclei in the present study are selected with Q_β greater than 3 MeV coming from a new compilation [33, 34]. All other branches are equated to one or two effective GT branches, and a logarithmic formula has been derived from Eq. (1) with the same notations as before in Eq. (3) in removing the term B_{F}.

$$\begin{aligned} -\ln(B_{\mathrm{GT}}) &= -\ln(\kappa) + \ln[\frac{2\pi\alpha Z_f}{1 - \exp(-2\pi\alpha Z_f)}] \\ &\ln(T_{1/2}) + \ln(\frac{1}{30}(E_0^5 - 10E_0^2 + 15E_0 - 6)). \end{aligned} \tag{3}$$

Now establish a linear formula describing the $-\ln(B_{\mathrm{GT}})$ based on physical meanings. After β-decay, an odd-odd (oo) nucleus decays to an even-even (ee) one while an odd-A (oA) one decays to another oA one. Q_β, calculated from ground state to ground state, does not necessarily reflect the actuality of E_0 of the effective branch. Thus, multiplicative factors c_1, c_2 and c_3 are introduced to include the odevity in the effective decay energy. Three equations are presented in taking the logarithm form with $Q_{\mathrm{oo}\to\mathrm{ee}}$, $Q_{\mathrm{oA}\to\mathrm{oA}}$ and $Q_{\mathrm{ee}\to\mathrm{oo}}$ denoting the effective decay energy for the different odevity cases.

$$\ln Q_\beta = \ln c_1 + \ln Q_{\mathrm{oo}\to\mathrm{ee}}, \tag{4}$$

$$\ln Q_\beta = \ln c_2 + \ln Q_{\mathrm{oA}\to\mathrm{oA}}, \tag{5}$$

$$\ln Q_\beta = \ln c_3 + \ln Q_{\mathrm{ee}\to\mathrm{oo}}. \tag{6}$$

Since the angular momentum of the ground state of the oo parent nucleus tends to differ significantly from that of the ground state of the ee daughter nucleus, the oo nucleus decays more likely to the excited state of the ee nucleus while the effective decay energy is small. With a similar analysis to the case of oA and ee nuclei, there is a pronounced stratification of reduced transition intensities of the three types owing to the different magnitudes of the multiplicative factors.

Combining Eq. (3) and the three equations above, a new formula is accordingly proposed for $-\ln(B_{\mathrm{GT}})$ with the Dirac function:

$$-\ln(B_{GT}) = a_0 + a_1\delta_{\mathrm{oA}} + a_2\delta_{\mathrm{oo}}. \tag{7}$$

The corresponding term for ee nuclei is set to a constant a_0. The Dirac functions δ_{oo} and δ_{oA} discern the odevity. Besides, the pronounced peaks of $-\ln(B_{\mathrm{GT}})$ values observed in the distribution of experimental data are bound up with the periodic arrangement of the nuclei, namely the shell effect of the nucleus. Therefore, a correction term a_3x_3 for the shell effect is added:

$$-\ln(B_{GT}) = a_0 + a_1\delta_{\mathrm{oA}} + a_2\delta_{\mathrm{oo}} + a_3x_3, \tag{8}$$

where a_0, a_1, a_2 and a_3 are fitting parameters, $x_3 = \sum_{N_k, Z_k} \exp[-(\frac{N-N_k}{5})^2 - (\frac{Z-Z_k}{5})^2]$, N_k and Z_k describe the shell and sub-shell structure and are chosen to be (50, 32) and (82, 50) according to where locate the distinct peak points.

Eqs. (7) and (8) can only describe a single decay branch without considering the βn branch. In the below content, βo denotes the branch other than βn. The relations between β-decay half-life and βn probability are shown as follows:

$$P_{\beta n} + P_{\beta\mathrm{o}} = 1, \tag{9}$$

$$\frac{1}{T_{1/2}} = \frac{1}{T_{\beta n}} + \frac{1}{T_{\beta\mathrm{o}}}, \tag{10}$$

$$-\ln P_{\beta n} = -\ln(\frac{T_{1/2}}{T_{\beta n}}), \tag{11}$$

$$-\ln(\frac{P_{\beta n}}{1 - P_{\beta n}}) = -\ln(\frac{T_{\beta\mathrm{o}}}{T_{\beta n}}), \tag{12}$$

where $T_{\beta n}$ ($T_{\beta\mathrm{o}}$) and $P_{\beta n}$ ($P_{\beta\mathrm{o}}$) denote the partial half-life and probability of βn (βo) branch. Thus, the analogical formula could be proposed for $-\ln(P_{\beta n})$ plus a new optimizing term for the effect of the difference between $Q_{\beta n}$ and Q_β on the βn probability:

$$-\ln(P_{\beta n}) = a_0 + a_1\delta_{\mathrm{oA}} + a_2\delta_{\mathrm{oo}} + a_3x_3 + a_4x_4, \tag{13}$$

where $x_4 = \ln(\frac{Q_{\beta n}}{m_e c^2}) - \ln(\frac{Q_{\beta\mathrm{o}}}{m_e c^2})$, $Q_{\beta n}$ ($Q_{\beta\mathrm{o}}$) denote the decay energy of βn (βo) branch from ground state to ground state. This makes use of a direct logarithmization of $P_{\beta n}$, yet it may obtain nonphysical results during the fitting, i.e. $P_{\beta n}$ > 1. Replacing $P_{\beta n}$ by $\frac{P_{\beta n}}{1 - P_{\beta n}}$ was used to avoid such results, and the derived formula is expressed as

$$-\ln(\frac{P_{\beta n}}{1 - P_{\beta n}}) = a_0 + a_1\delta_{\mathrm{oA}} + a_2\delta_{\mathrm{oo}} + a_3x_3 + a_4x_4. \tag{14}$$

B. Bootstrap method

The present work evaluates the fitting parameters and uncertainties of the β^--decay half-lives and βn probabilities by applying the bootstrap method. In applied statistics, the bootstrap method was proposed by Efron Bradley in 1985 [37] and used to determine the accuracy of estimating unknown parameters of a chosen estimator through the basic idea of resampling [38].

After the first proposal of application on alpha decay laws [38], it was successfully applied to the uncertainty determination of the nuclear mass models [39, 40], the proton(s) decay stability [40], and the binary cluster model [41].

Similar to Monte Carlo events, a new dataset is obtained by resampling with replacements from a given experimental dataset. A group of parameters in the model is obtained by minimizing the root-mean-square of residuals. Repeating this process M times, the total uncertainty is thus

$$\sigma_{\text{tot}} = \sqrt{\frac{1}{MK}\sum_{m,k}(r_{m,k})^2}, \tag{15}$$

where m denotes the m^{th} resampling of the dataset and k denotes the k^{th} nucleus in the original dataset among the K nuclei. $r_{m,k}$ is the residual between the observed value and its estimated value from the m^{th} replication of bootstrap for the k^{th} nucleus.

The total uncertainty is then decomposed into the statistical one σ_{stat} and the systematic one σ_{sys}, and the two are written as:

$$\sigma_{\text{stat}} = \sqrt{\frac{1}{(M-1)K}\sum_{m,k}(r_{m,k}-\bar{r}_k)^2}, \tag{16}$$

$$\sigma_{\text{sys}} = \sqrt{\frac{1}{K}\sum_{k}(\frac{1}{M}\sum_{m} r_{m,k})^2}. \tag{17}$$

In the present work, this method is used to assess the uncertainty of Eqs. (7), (8), (13), and (14). The global systematic uncertainty of model deficiency σ_{sys} and the statistical uncertainty proper to the specific nucleus are combined to evaluate the confidence interval:

$$\sigma_{\text{pred},k} = \sqrt{\sigma^2_{\text{stat},k} + \sigma^2_{\text{sys}}}. \tag{18}$$

As the reduced transition probability term and the probabilities of βn and βo branches are in logarithmic form, the uncertainties obtained are propagated to be different positive and negative deviations when the half-lives and the probabilities are further calculated.

III. RESULTS AND DISCUSSIONS

This section investigates the half-life of β^- decay and the probability of occurrence of βn decay with two experimental datasets and several formulas proposed before. The bootstrap method is brought to deal with the uncertainty analysis.

A. One effective decay branch

From the newest compilation and evaluation, the focused nuclei are carefully selected with experimental half-lives smaller than 2 s and Q greater than 3 MeV, because the forbidden transitions may dominate long-lived transitions and are not suitable to be studied as effective GT branches. The nuclei having βn basically meet these two conditions. Among 256 selected nuclei, two datasets were taken: one contains all 256, which have half-life measurements, and the other contains 133, which have βn probabilities measurements.

TABLE 1. Mean value of fitting parameters corresponding to Eq. (7) ($1b3p$) and Eq. (8) ($1b4p$) in using the first dataset. $1b3p$ and $1b4p$ denote one branch assumption with three and four parameters in the fitting formulas (7) and (8), respectively. The three columns σ^2_{stat}, σ^2_{sys} and σ^2_{tot} exhibit the square of statistical, systematic and total uncertainties.

Type	Fitting parameters				Uncertainties		
	a_0	a_1	a_2	a_3	σ^2_{stat}	σ^2_{sys}	σ^2_{tot}
$1b3p$	0.8938	1.0489	1.9296	-	0.0059	0.4887	0.4946
$1b4p$	0.5053	1.0450	1.9081	0.9459	0.0053	0.3226	0.3278

Based on the assumption of one effective decay, the bootstrap method is used to investigate the first dataset, as listed in TABLE 1. The notations, $1b3p$ and $1b4p$, denote one branch assumption with three parameters in (7) and four parameters in (8), respectively. The shell effect does contribute to the description of half-lives, as seen from the decreasing systematic uncertainty. Fig. 1 shows the results of the difference between the observed data and calculation corresponding to Eq. (7) ($1b3p$) and Eq. (8) ($1b4p$). The latter with the shell effect reproduces better than the former in general, especially where the major shell and sub-shell closures are located.

B. Two effective decay branches

Then consider the case where there are neutron emissions after β decay, i.e. two decay branches with βn emitting neutrons and βo without emitting. It should be mentioned that there is no distinction between the number of neutrons emitted. Eq. (8) is applied to each branch with the second dataset. The corresponding results are listed in the last two lines of TABLE 2 with the notations $(2b4p)_{\beta n}$ and $(2b4p)_{\beta o}$, respectively.

TABLE 2. Mean value of fitting parameters corresponding to Eq. (8) with four parameters and one branch assumption ($1b4p$), and with the two decay branches separately ($(2b4p)_{\beta n}$ and $(2b4p)_{\beta o}$) in using the second dataset. The three columns σ^2_{stat}, σ^2_{sys} and σ^2_{tot} exhibit the square of statistical, systematic and total uncertainties.

Type	Fitting parameters				Uncertainties		
	a_0	a_1	a_2	a_3	σ^2_{stat}	σ^2_{sys}	σ^2_{tot}
$1b4p$	0.3649	1.0179	2.0070	1.0725	0.0079	0.2565	0.2644
$(2b4p)_{\beta n}$	0.4384	0.6672	0.9952	0.3666	0.0255	0.8433	0.8687
$(2b4p)_{\beta o}$	0.6838	1.1007	1.9844	1.4825	0.0131	0.4428	0.4559

Moreover, one branch result has been re-fitted to compare the adaptability of Eq. (8) with the changed dataset. The val-

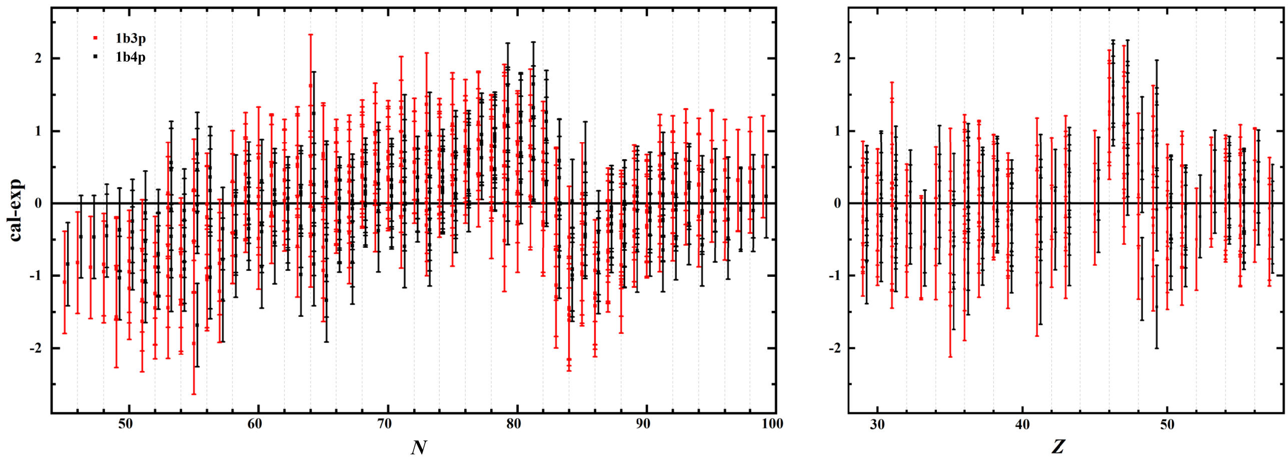

Fig. 1. Distribution of the difference between the calculated values and the experimental values of $\ln T$ with error bars. The red squares and the black squares, in the one-branch case, correspond to the Eq. (7) with three parameters ($1b3p$) and Eq. (8) with four parameters ($1b4p$), respectively.

ues of βn probability largely vary from nuclide to nuclide, resulting larger uncertainties in $(2b4p)_{\beta n}$ and $(2b4p)_{\beta o}$ compared with $1b4p$.

The values of $-\ln(B_{GT})$ were calculated to deduce the partial half-life of the two branches $T_{\beta n}$, and $T_{\beta o}$, then the $T_{1/2}$ appearing in the Eqs. (10) and (11), and also the branching ratios. The experimental half-lives are generally well reproduced, in particular, the shorter the half-life, the better the consistency.

The root-mean-square deviation (RMS), characterizing the goodness-of-fit for one dataset, is defined as:

$$\mathrm{RMS} = \frac{1}{K}\sum_{k=1}^{K}[Y_{k,cal} - Y_{k,exp}]^2. \tag{19}$$

The results of directly fitting to βn probability are listed in TABLE 3. The three results and experimental values are generally consistent within about 10% RMS (10.389%, 10.002%, and 10.231%, respectively).

TABLE 3. Mean value of fitting parameters corresponding to Eq. (13) ($\ln p$) and Eq. (14) ($\ln p1p$) in using the second dataset. The three columns σ^2_{stat}, σ^2_{sys} and σ^2_{tot} exhibit the square of statistical, systematic and total uncertainties.

Type	Fitting parameters					Uncertainties		
	a_0	a_1	a_2	a_3	a_4	σ^2_{stat}	σ^2_{sys}	σ^2_{tot}
$\ln p$	0.1956	-0.2570	-0.7434	-0.7137	-3.9323	0.0287	0.6966	0.7253
$\ln p1p$	-0.2697	-0.3780	-0.8663	-1.0562	-4.4410	0.0307	0.7492	0.7800

The one given by $-\ln(P_{\beta n})$ is close to the experimental value in general. But the $-\ln(\frac{P_{\beta n}}{1-P_{\beta n}})$ results are closer to the experimental data in some cases with great undulations, such as in the regions of shell closure. The form of $-\ln(\frac{P_{\beta n}}{1-P_{\beta n}})$ determines in a more physical transformation, guaranteeing the calculated probabilities always between 0 and 1.

The experimental and calculated values of the half-life and the probability are shown in the TABLE 4 for the two-branch case. The calculated values of the half-life come from the fitting results of $(2b4p)_{\beta n}$ and $(2b4p)_{\beta o}$ listed in the TABLE 2, while the probability corresponds to Eq. (13) ($\ln p$) listed in the TABLE 3.

Since the uncertainties are estimated on a logarithmic scale, the distances from the upper and lower bounds to the predicted values are different when converting, and thus there are differences in the positive and negative directions of the uncertainties.

For the sake of convenience, the half-lives are presented in natural logarithm form with T in unit of seconds so that the total uncertainty of calculated values could be given in the same scale in the sixth column. Since the two formulas Eqs. (13) and (14) study different objects, the calculated values of probability are converted from the natural logarithm form.

C. Predictions

According to the presented method, the predictions of β-decay half-lives and βn probabilities $P_{\beta n}$ could be given for the nuclei without experimental data, as well as the uncertainties according to the Eq. (18). The predictions of neutron-rich nuclei in the intermediate mass zone furnish important nuclear input and relevant data for nuclear physics applications, such as the fission products yield in nuclear reactors [42], and the half-lives of nuclei participating in the rapid neutron capture process (r process) in astrophysics [43].

In TABLE 5, 123 nuclei without experimental values of $P_{\beta n}$ are listed, 18 of which furthermore have no half-life (in the last eighteen lines of the table). The predictions corre-

sponding to Eq. (14) and Eq. (8) are given for the probability and half-life, respectively.

TABLE 4. The experimental and calculated values of the half-life and the probability for the two-branch case. Q_β and $Q_{\beta n}$ are the decay energy of β-decay and βn-decay in keV. $\ln T_{cal}$ and P_{cal} are calculated using Eqs. (8) and (14). The uncertainties are given in one standard deviation estimated with bootstrap method.

Nucl.	Q_β	$Q_{\beta n}$	$\ln T_{exp}$	$\ln T_{cal}$	$\sigma_{\ln T_{cal}}$	$P_{exp}(\%)$	$P_{cal}(\%)$
$^{74}_{29}$Cu	9750.5	1515.9	0.469	0.254	0.573	0.075	$0.17^{-0.10}_{+0.24}$
$^{75}_{29}$Cu	8087.6	3214.1	0.202	0.33	0.575	2.7	$6.84^{-3.88}_{+8.19}$
$^{76}_{29}$Cu	11327	3511.6	-0.451	-0.232	0.575	7.2	$4.16^{-2.39}_{+5.29}$
$^{77}_{29}$Cu	10170	5610	-0.755	-0.766	0.574	30.1	$26.95^{-13.61}_{+19.98}$
$^{78}_{29}$Cu	12990	6220	-1.104	-0.969	0.573	51	$25.10^{-12.84}_{+19.46}$
$^{79}_{29}$Cu	11690	7670	-1.422	-1.647	0.573	66	$45.82^{-19.68}_{+21.08}$
$^{80}_{29}$Cu	15450	9160	-2.178	-2.143	0.572	58	$46.40^{-19.82}_{+21.02}$
$^{81}_{29}$Cu	14780	12160	-2.615	-3.428	0.574	81	$68.53^{-20.77}_{+15.30}$
$^{79}_{30}$Zn	9115.4	2202.3	-0.293	0.102	0.58	1.75	$0.98^{-0.58}_{+1.41}$
$^{80}_{30}$Zn	7575.1	2827.8	-0.576	-0.153	0.584	1.36	$4.52^{-2.60}_{+5.77}$
$^{81}_{30}$Zn	11428.3	4952.7	-1.205	-1.08	0.578	18	$11.75^{-6.52}_{+12.56}$
$^{82}_{30}$Zn	10616.8	7242.7	-1.727	-2.234	0.579	69	$40.15^{-18.30}_{+21.53}$
$^{84}_{30}$Zn	12160	9260	-2.926	-3.177	0.579	73	$50.33^{-20.59}_{+20.47}$
$^{80}_{31}$Ga	10311.6	2232	0.642	0.388	0.578	0.9	$0.99^{-0.58}_{+1.38}$
$^{81}_{31}$Ga	8663.7	3836	0.196	0.209	0.577	12.5	$12.85^{-7.09}_{+13.37}$
$^{82}_{31}$Ga	12484.3	5289.6	-0.509	-0.688	0.575	22.7	$16.48^{-8.89}_{+15.67}$
$^{83}_{31}$Ga	11719.3	8086.6	-1.171	-1.814	0.573	67	$51.24^{-20.68}_{+20.26}$
$^{84}_{31}$Ga	14060	8820	-2.352	-1.859	0.572	47.6	$52.66^{-20.89}_{+20.00}$
$^{85}_{31}$Ga	13270	10230	-2.387	-2.72	0.574	81.3	$62.76^{-21.34}_{+17.31}$
$^{86}_{31}$Ga	15320	10970	-3.012	-2.666	0.573	85.2	$66.10^{-21.11}_{+16.20}$
$^{87}_{31}$Ga	14830	12080	-3.54	-3.448	0.574	91.2	$67.57^{-20.91}_{+15.66}$
$^{84}_{32}$Ge	7705.1	3449.6	-0.049	-0.32	0.582	10.6	$9.60^{-5.39}_{+10.82}$
$^{85}_{32}$Ge	10065.7	4658.8	-0.699	-0.542	0.577	16.2	$15.22^{-8.28}_{+14.96}$
$^{86}_{32}$Ge	9560	5720	-1.505	-1.55	0.579	45	$27.79^{-14.00}_{+20.29}$
$^{86}_{33}$As	11541	5380.2	-0.058	-0.449	0.573	34.5	$22.93^{-11.90}_{+18.72}$
$^{87}_{33}$As	10808.2	6813.9	-0.726	-1.293	0.573	15.4	$40.82^{-18.43}_{+21.43}$
$^{88}_{34}$Se	6831.8	1936.2	0.412	0.148	0.583	0.99	$1.24^{-0.73}_{+1.73}$
$^{89}_{34}$Se	9281.9	3652.3	-0.821	-0.323	0.576	7.8	$6.95^{-3.95}_{+8.32}$
$^{91}_{34}$Se	10530	5350	-1.309	-1.207	0.572	21	$16.97^{-9.11}_{+15.91}$
$^{90}_{35}$Br	10959	4464.2	0.648	-0.517	0.571	25.6	$10.76^{-5.97}_{+11.66}$
$^{91}_{35}$Br	9866.7	5780.6	-0.609	-1.216	0.57	30.4	$25.02^{-12.77}_{+19.35}$
$^{92}_{35}$Br	12536.5	6669.8	-1.097	-1.619	0.569	33.1	$24.14^{-12.41}_{+19.11}$
$^{93}_{35}$Br	11250	7810	-1.884	-2.298	0.568	64	$36.52^{-17.10}_{+21.34}$
$^{94}_{35}$Br	13950	8670	-2.659	-2.548	0.568	30	$34.36^{-16.43}_{+21.27}$
$^{92}_{36}$Kr	6003.1	904.4	0.61	-0.022	0.578	0.0332	$0.04^{-0.03}_{+0.07}$
$^{93}_{36}$Kr	8483.9	2565.1	0.25	-0.716	0.569	1.99	$1.26^{-0.74}_{+1.74}$
$^{94}_{36}$Kr	7215	3201	-1.556	-1.218	0.577	1.11	$4.17^{-2.41}_{+5.37}$
$^{95}_{36}$Kr	9733	4333	-2.172	-1.643	0.569	2.87	$5.66^{-3.22}_{+6.94}$
$^{96}_{36}$Kr	8275	4741	-2.526	-2.098	0.576	3.7	$10.70^{-5.99}_{+11.82}$
$^{97}_{36}$Kr	11100	5860	-2.779	-2.436	0.568	6.7	$10.51^{-5.84}_{+11.47}$
$^{98}_{36}$Kr	10060	6140	-3.147	-3.123	0.576	7	$13.05^{-7.23}_{+13.65}$
$^{99}_{36}$Kr	12360	7540	-3.297	-3.076	0.568	11	$17.75^{-9.49}_{+16.34}$
$^{95}_{37}$Rb	9228	4883	-0.973	-1.533	0.568	8.8	$10.94^{-6.07}_{+11.81}$
$^{96}_{37}$Rb	11569.8	5693.9	-1.601	-1.799	0.569	14.1	$12.23^{-6.76}_{+12.89}$
$^{97}_{37}$Rb	10062.3	6333.7	-1.778	-2.133	0.568	24.9	$19.96^{-10.53}_{+17.44}$
$^{98}_{37}$Rb	12054	6141	-2.163	-2.056	0.57	14.354	$13.58^{-7.46}_{+13.88}$
$^{99}_{37}$Rb	11400.3	7230.6	-2.851	-2.752	0.568	19.1	$20.25^{-10.67}_{+17.58}$
$^{100}_{37}$Rb	13574	8203	-2.976	-2.775	0.569	5.75	$24.98^{-12.83}_{+19.51}$
$^{101}_{37}$Rb	12480	8910	-3.474	-3.318	0.568	28	$29.98^{-14.81}_{+20.63}$
$^{102}_{37}$Rb	14450	9550	-3.297	-3.195	0.569	18	$33.11^{-16.04}_{+21.23}$
$^{97}_{38}$Sr	7540	1683.1	-0.846	-0.594	0.569	0.03	$0.25^{-0.15}_{+0.36}$
$^{98}_{38}$Sr	5871.7	1627	-0.426	-0.543	0.579	0.23	$0.44^{-0.26}_{+0.63}$
$^{99}_{38}$Sr	8128.4	1702.1	-1.313	-0.959	0.569	0.096	$0.18^{-0.11}_{+0.27}$
$^{100}_{38}$Sr	7506	2757.6	-1.606	-1.699	0.578	1.11	$1.51^{-0.89}_{+2.10}$
$^{101}_{38}$Sr	9736	3931	-2.163	-1.842	0.569	2.52	$3.29^{-1.90}_{+4.29}$
$^{102}_{38}$Sr	9014	4830	-2.631	-2.618	0.577	5.5	$7.58^{-4.32}_{+9.04}$
$^{98}_{39}$Y	8992	2576.6	-0.601	-0.578	0.571	0.33	$1.20^{-0.70}_{+1.65}$

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Nucl.	Q_β	$Q_{\beta n}$	$\ln T_{exp}$	$\ln T_{cal}$	$\sigma_{\ln T_{cal}}$	$P_{exp}(\%)$	$P_{cal}(\%)$
$^{99}_{39}$Y	6971	2566	0.391	-0.277	0.569	1.97	$2.21^{-1.28}_{+2.97}$
$^{100}_{39}$Y	9050	2222	-0.312	-0.605	0.571	1.02	$0.61^{-0.36}_{+0.85}$
$^{101}_{39}$Y	8105	3245	-0.839	-0.992	0.569	1.98	$3.18^{-1.83}_{+4.15}$
$^{102}_{39}$Y	10414.5	3921.5	-1.204	-1.303	0.571	4	$3.91^{-2.25}_{+5.03}$
$^{103}_{39}$Y	9358	5059	-1.444	-1.749	0.568	8.1	$11.07^{-6.14}_{+11.93}$
$^{104}_{39}$Y	11660	5680	-1.551	-1.918	0.57	34	$11.32^{-6.29}_{+12.21}$
$^{106}_{41}$Nb	9931	3062.5	0.013	-1.088	0.571	4.5	$1.65^{-0.96}_{+2.24}$
$^{107}_{41}$Nb	8828	4339	-1.248	-1.47	0.569	7.4	$7.54^{-4.26}_{+8.84}$
$^{108}_{41}$Nb	11210	4934	-1.65	-1.723	0.57	6.3	$7.53^{-4.26}_{+8.87}$
$^{109}_{41}$Nb	9980	5990	-2.18	-2.147	0.568	31	$16.53^{-8.90}_{+15.66}$
$^{110}_{41}$Nb	12230	6280	-2.59	-2.207	0.57	40	$13.90^{-7.62}_{+14.11}$
$^{109}_{42}$Mo	7617	1185	-0.368	-0.719	0.569	1.3	$0.05^{-0.03}_{+0.07}$
$^{110}_{42}$Mo	6492	1669	-1.248	-1.077	0.579	2	$0.31^{-0.19}_{+0.45}$
$^{109}_{43}$Tc	6456	1307	-0.117	0.035	0.569	0.08	$0.16^{-0.09}_{+0.23}$
$^{110}_{43}$Tc	9038	1633	-0.093	-0.659	0.571	0.04	$0.16^{-0.09}_{+0.23}$
$^{111}_{43}$Tc	7761	2977	-1.224	-0.848	0.569	0.85	$2.64^{-1.53}_{+3.50}$
$^{112}_{43}$Tc	10372	3455	-1.187	-1.333	0.571	1.7	$2.31^{-1.34}_{+3.08}$
$^{113}_{43}$Tc	9057	4748	-1.884	-1.646	0.568	2.1	$9.79^{-5.47}_{+10.88}$
$^{114}_{43}$Tc	11620	5200	-2.408	-1.932	0.57	1.3	$8.06^{-4.55}_{+9.38}$
$^{118}_{45}$Rh	10501	3466	-1.255	-1.366	0.57	3.1	$2.31^{-1.34}_{+3.08}$
$^{119}_{45}$Rh	8585	4494.6	-1.661	-1.318	0.568	6.4	$10.48^{-5.83}_{+11.44}$
$^{123}_{46}$Pd	9120	2610	-2.226	-0.862	0.571	10	$1.27^{-0.74}_{+1.76}$
$^{124}_{46}$Pd	7810	3090	-2.513	-1.056	0.579	17	$4.12^{-2.37}_{+5.29}$
$^{125}_{46}$Pd	10400	4010	-2.813	-1.135	0.575	12	$6.03^{-3.44}_{+7.37}$
$^{126}_{46}$Pd	8820	4590	-3.024	-1.396	0.579	22	$15.74^{-8.55}_{+15.31}$
$^{121}_{47}$Ag	6671	1483	-0.252	0.408	0.57	0.08	$0.36^{-0.22}_{+0.53}$
$^{122}_{47}$Ag	9506	1896	-0.637	-0.232	0.571	0.186	$0.41^{-0.24}_{+0.57}$
$^{123}_{47}$Ag	7866	2993	-1.217	-0.113	0.572	0.56	$4.68^{-2.68}_{+5.89}$
$^{124}_{47}$Ag	10500	3140	-1.655	-0.428	0.573	1.3	$2.97^{-1.71}_{+3.87}$
$^{125}_{47}$Ag	8830	4110	-1.833	-0.449	0.573	11.8	$12.9^{-7.09}_{+13.33}$
$^{127}_{47}$Ag	10310	5750	-2.477	-1.145	0.574	14.6	$27.92^{-14.01}_{+20.22}$

Continued on next column

Nucl.	Q_β	$Q_{\beta n}$	$\ln T_{exp}$	$\ln T_{cal}$	$\sigma_{\ln T_{cal}}$	$P_{exp}(\%)$	$P_{cal}(\%)$
$^{128}_{47}$Ag	12620	6060	-2.813	-1.136	0.573	20	$25.34^{-12.94}_{+19.53}$
$^{130}_{48}$Cd	8766	3649	-2.064	-1.163	0.583	3.5	$7.13^{-4.06}_{+8.54}$
$^{131}_{48}$Cd	12810	6590	-2.489	-2.043	0.575	3.5	$22.19^{-11.58}_{+18.46}$
$^{132}_{48}$Cd	12150	9690	-2.477	-3.522	0.582	60	$57.31^{-21.35}_{+18.93}$
$^{128}_{49}$In	9220	1250	-0.174	0.614	0.578	0.0384	$0.12^{-0.07}_{+0.18}$
$^{129}_{49}$In	7753	2453	-0.496	0.528	0.58	0.23	$3.19^{-1.85}_{+4.24}$
$^{130}_{49}$In	10249	2636	-1.291	0.11	0.578	1.2	$2.1^{-1.22}_{+2.84}$
$^{131}_{49}$In	9239.5	4035.9	-1.324	-0.382	0.577	2.21	$12.19^{-6.75}_{+12.89}$
$^{132}_{49}$In	14135	6782	-1.603	-1.691	0.573	12.3	$25.53^{-13.02}_{+19.59}$
$^{133}_{49}$In	13410	11010	-1.82	-3.251	0.575	90	$69.46^{-20.61}_{+14.96}$
$^{133}_{50}$Sn	8049.6	690.1	0.351	0.38	0.581	0.0294	$0.01^{-0.01}_{+0.02}$
$^{134}_{50}$Sn	7586.8	4418.4	-0.103	-0.73	0.579	17	$25.45^{-13.03}_{+19.66}$
$^{135}_{50}$Sn	9058.1	5317	-0.662	-0.589	0.574	21	$34.03^{-16.31}_{+21.23}$
$^{136}_{50}$Sn	8610	5720	-1.064	-1.503	0.579	27	$37.98^{-17.66}_{+21.53}$
$^{137}_{50}$Sn	10270	6650	-1.444	-1.354	0.573	50	$44.37^{-19.35}_{+21.22}$
$^{135}_{51}$Sb	8038.5	4772.1	0.512	-0.071	0.573	20	$35.01^{-16.64}_{+21.32}$
$^{136}_{51}$Sb	9918.4	5150.6	-0.079	-0.159	0.572	25.14	$32.46^{-15.75}_{+21.06}$
$^{137}_{51}$Sb	9243	6294	-0.681	-1.006	0.573	49	$49.27^{-20.36}_{+20.60}$
$^{138}_{51}$Sb	11500	7000	-1.1	-1.18	0.571	72	$48.66^{-20.26}_{+20.71}$
$^{139}_{51}$Sb	10420	7840	-1.704	-1.85	0.573	90	$59.42^{-21.37}_{+18.31}$
$^{140}_{51}$Sb	12640	8200	-1.772	-1.816	0.571	30.6	$54.76^{-21.12}_{+19.53}$
$^{138}_{52}$Te	6283.9	2589	0.378	0.195	0.581	4.82	$6.21^{-3.54}_{+7.59}$
$^{140}_{53}$I	9380	3967	-0.528	-0.085	0.571	7.88	$12.47^{-6.86}_{+12.98}$
$^{141}_{53}$I	8271	4988	-0.868	-0.707	0.569	21.2	$27.50^{-13.80}_{+20.05}$
$^{141}_{54}$Xe	6280.2	781.5	0.547	0.779	0.572	0.0433	$0.03^{-0.02}_{+0.05}$
$^{142}_{54}$Xe	5284.9	1176.6	0.2	0.278	0.578	0.36	$0.25^{-0.15}_{+0.36}$
$^{143}_{54}$Xe	7472.6	2240.4	-0.671	-0.402	0.569	1	$1.21^{-0.71}_{+1.68}$
$^{144}_{54}$Xe	6399	2731.9	-0.947	-0.937	0.577	3	$3.54^{-2.05}_{+4.63}$
$^{145}_{54}$Xe	8561	3707	-1.671	-1.314	0.569	5	$5.04^{-2.88}_{+6.27}$
$^{146}_{54}$Xe	7355	4028	-1.924	-1.811	0.576	6.9	$8.93^{-5.04}_{+10.29}$
$^{142}_{55}$Cs	7327.7	1146.8	0.523	0.692	0.57	0.0916	$0.12^{-0.07}_{+0.18}$

Continued on next column

Nucl.	Q_β	$Q_{\beta n}$	$\ln T_{exp}$	$\ln T_{cal}$	$\sigma_{\ln T_{cal}}$	$P_{exp}(\%)$	$P_{cal}(\%)$
$^{143}_{55}$Cs	6261.7	2095.4	0.585	0.328	0.569	1.582	$1.86^{-1.08}_{+2.52}$
$^{144}_{55}$Cs	8496	2595	-0.011	-0.357	0.57	2.98	$1.83^{-1.06}_{+2.47}$
$^{145}_{55}$Cs	7462	3641	-0.541	-0.784	0.568	13.5	$7.89^{-4.44}_{+9.16}$
$^{146}_{55}$Cs	9637	4134.5	-1.134	-1.155	0.57	14.3	$7.04^{-3.99}_{+8.38}$
$^{147}_{55}$Cs	8344	4956	-1.472	-1.485	0.568	28.5	$16.16^{-8.72}_{+15.44}$
$^{148}_{55}$Cs	10683	5282	-1.89	-1.744	0.57	29	$12.09^{-6.69}_{+12.80}$
$^{149}_{55}$Cs	9870	6270	-2.235	-2.349	0.568	25	$20.36^{-10.72}_{+17.63}$
$^{150}_{55}$Cs	11730	6880	-2.513	-2.326	0.569	20	$22.58^{-11.78}_{+18.67}$
$^{147}_{56}$Ba	6414	715	-0.112	-0.115	0.569	0.066	$0.01^{-0.01}_{+0.02}$
$^{148}_{56}$Ba	5115	1013	-0.483	-0.184	0.579	0.39	$0.10^{-0.06}_{+0.15}$
$^{149}_{56}$Ba	7100	1520	-1.044	-0.602	0.569	2.2	$0.20^{-0.12}_{+0.30}$
$^{148}_{57}$La	7690	1234	0.293	-0.104	0.571	0.19	$0.09^{-0.05}_{+0.13}$
$^{149}_{57}$La	6450	2110	0.087	-0.182	0.569	1.41	$1.32^{-0.77}_{+1.82}$
$^{150}_{57}$La	8720	2470	-0.673	-0.709	0.571	2.69	$1.14^{-0.66}_{+1.57}$

TABLE 5. The predictions of the half-life and probability for 123 nuclei corresponding to Eq. (8) and Eq. (14). The experimental and predicted half-lives are in logarithmic scale. The uncertainties are given in one standard deviation estimated with bootstrap method.

Nucl.	Q_β	$Q_{\beta n}$	$\ln T_{exp}$	$\ln T_{pred}$	$\sigma_{\ln T_{pred}}$	$P_{cal}(\%)$
$^{82}_{29}$Cu	16990	12810	-3.433	-2.444	0.517	$69.09^{-21.29}_{+15.36}$
$^{78}_{30}$Zn	6222.7	437.8	0.385	0.750	0.522	0
$^{87}_{32}$Ge	11540	6810	-2.271	-1.383	0.514	$34.71^{-16.65}_{+21.53}$
$^{88}_{32}$Ge	10580	7410	-2.800	-1.914	0.523	$43.62^{-19.54}_{+21.78}$
$^{88}_{33}$As	13160	7640	-1.609	-1.154	0.518	$43.59^{-19.41}_{+21.61}$
$^{90}_{34}$Se	8200	4400	-1.635	-0.886	0.520	$16.12^{-8.70}_{+15.56}$
$^{100}_{36}$Kr	11200	8000	-4.962	-3.289	0.518	$22.99^{-12.03}_{+19.20}$
$^{103}_{37}$Rb	13810	10480	-3.772	-3.395	0.511	$36.03^{-17.08}_{+21.58}$
$^{96}_{38}$Sr	5411.7	213	0.067	0.333	0.518	0
$^{103}_{38}$Sr	11040	5680	-2.937	-2.292	0.511	$9.15^{-5.09}_{+10.29}$
$^{104}_{38}$Sr	9960	6280	-2.937	-2.743	0.518	$14.60^{-7.95}_{+14.74}$
$^{105}_{38}$Sr	12700	7380	-3.244	-2.993	0.511	$14.73^{-7.99}_{+14.57}$

Continued on next column

Nucl.	Q_β	$Q_{\beta n}$	$\ln T_{exp}$	$\ln T_{pred}$	$\sigma_{\ln T_{pred}}$	$P_{cal}(\%)$
$^{106}_{38}$Sr	11260	8400	-3.912	-3.356	0.518	$26.42^{-13.56}_{+20.39}$
$^{106}_{39}$Y	12500	7340	-2.501	-1.986	0.513	$22.86^{-11.91}_{+18.93}$
$^{107}_{39}$Y	12000	8110	-3.396	-2.726	0.511	$25.22^{-12.89}_{+19.57}$
$^{108}_{39}$Y	14060	9000	-3.507	-2.574	0.513	$30.26^{-15.07}_{+21.09}$
$^{109}_{39}$Y	12990	10080	-3.689	-3.122	0.511	$38.33^{-17.83}_{+21.68}$
$^{108}_{40}$Zr	8190	4300	-2.545	-1.797	0.518	$7.06^{-3.97}_{+8.44}$
$^{109}_{40}$Zr	10500	5280	-2.882	-2.075	0.511	$8.34^{-4.66}_{+9.56}$
$^{110}_{40}$Zr	9400	5730	-3.283	-2.486	0.518	$12.83^{-7.04}_{+13.48}$
$^{111}_{40}$Zr	11320	6700	-3.730	-2.451	0.511	$15.79^{-8.51}_{+15.24}$
$^{112}_{40}$Zr	10460	6990	-3.507	-3.021	0.518	$18.1^{-9.70}_{+16.90}$
$^{103}_{41}$Nb	5932	466.1	0.307	0.764	0.511	0
$^{111}_{41}$Nb	11060	7600	-2.919	-2.351	0.511	$26.64^{-13.50}_{+19.99}$
$^{112}_{41}$Nb	13190	7600	-3.270	-2.288	0.513	$21.42^{-11.24}_{+18.33}$
$^{113}_{41}$Nb	11980	8880	-3.442	-2.750	0.511	$33.67^{-16.25}_{+21.37}$
$^{114}_{41}$Nb	14420	9030	-4.075	-2.734	0.513	$28.25^{-14.25}_{+20.65}$
$^{115}_{41}$Nb	13400	10380	-3.772	-3.310	0.511	$38.15^{-17.78}_{+21.68}$
$^{112}_{42}$Mo	7800	3490	-2.079	-1.586	0.518	$3.60^{-2.05}_{+4.66}$
$^{113}_{42}$Mo	10320	4700	-2.526	-2.021	0.511	$5.55^{-3.14}_{+6.79}$
$^{114}_{42}$Mo	8790	4930	-2.847	-2.184	0.518	$9.24^{-5.15}_{+10.52}$
$^{115}_{42}$Mo	11570	5780	-3.090	-2.592	0.511	$8.13^{-4.54}_{+9.36}$
$^{116}_{42}$Mo	9960	6750	-3.442	-2.807	0.518	$19.06^{-10.17}_{+17.41}$
$^{117}_{42}$Mo	12210	7210	-3.817	-2.855	0.511	$15.74^{-8.49}_{+15.21}$
$^{118}_{42}$Mo	11160	7680	-3.963	-3.351	0.518	$20.52^{-10.87}_{+18.12}$
$^{115}_{43}$Tc	9870	5830	-2.551	-1.813	0.511	$15.67^{-8.45}_{+15.17}$
$^{116}_{43}$Tc	12610	6660	-2.865	-2.093	0.513	$15.66^{-8.47}_{+15.27}$
$^{117}_{43}$Tc	11110	7620	-3.112	-2.399	0.511	$26.61^{-13.48}_{+19.98}$
$^{118}_{43}$Tc	13470	7630	-3.507	-2.403	0.513	$20.54^{-10.83}_{+17.92}$
$^{119}_{43}$Tc	12190	8820	-3.817	-2.808	0.511	$32.68^{-15.88}_{+21.22}$
$^{120}_{43}$Tc	14490	8980	-3.863	-2.645	0.513	$30.35^{-15.07}_{+21.02}$
$^{121}_{43}$Tc	13270	10160	-3.817	-3.001	0.510	$44.15^{-19.42}_{+21.40}$
$^{114}_{44}$Ru	5489	474.4	-0.611	0.139	0.518	0

Continued on next column

Nucl.	Q_β	$Q_{\beta n}$	$\ln T_{exp}$	$\ln T_{pred}$	$\sigma_{\ln T_{pred}}$	$P_{cal}(\%)$
$^{115}_{44}$Ru	8040	1450	-1.146	-0.803	0.511	$0.10^{-0.06}_{+0.14}$
$^{116}_{44}$Ru	6667	2089.6	-1.590	-0.830	0.518	$0.77^{-0.44}_{+1.07}$
$^{117}_{44}$Ru	9410	3170	-1.890	-1.582	0.511	$1.54^{-0.88}_{+2.08}$
$^{118}_{44}$Ru	7630	3570	-2.313	-1.482	0.518	$4.47^{-2.54}_{+5.66}$
$^{119}_{44}$Ru	10260	4250	-2.666	-1.962	0.511	$3.95^{-2.25}_{+5.01}$
$^{120}_{44}$Ru	8800	4740	-3.101	-2.092	0.518	$8.84^{-4.93}_{+10.12}$
$^{121}_{44}$Ru	11200	5700	-3.540	-2.218	0.510	$10.96^{-6.05}_{+11.79}$
$^{122}_{44}$Ru	9930	6030	-3.689	-2.416	0.518	$17.97^{-9.61}_{+16.66}$
$^{123}_{44}$Ru	12280	6930	-3.963	-2.310	0.510	$22.13^{-11.51}_{+18.39}$
$^{124}_{44}$Ru	10930	7330	-4.200	-2.489	0.520	$33.99^{-16.47}_{+21.61}$
$^{115}_{45}$Rh	6197	1190	0.030	0.485	0.511	$0.13^{-0.08}_{+0.19}$
$^{121}_{45}$Rh	9930	5960	-2.577	-1.632	0.510	$20.36^{-10.70}_{+17.63}$
$^{122}_{45}$Rh	12540	6030	-2.976	-1.721	0.513	$15.22^{-8.23}_{+14.88}$
$^{123}_{45}$Rh	11070	7190	-3.170	-1.877	0.510	$33.01^{-15.96}_{+21.18}$
$^{124}_{45}$Rh	13500	7470	-3.507	-1.751	0.515	$32.14^{-15.69}_{+21.18}$
$^{125}_{45}$Rh	12120	8320	-3.631	-1.997	0.512	$46.99^{-20.16}_{+21.}$
$^{126}_{45}$Rh	14560	8750	-3.963	-1.858	0.518	$47.36^{-20.31}_{+21.23}$
$^{127}_{45}$Rh	13150	9760	-3.912	-2.252	0.514	$59.36^{-21.77}_{+18.60}$
$^{119}_{46}$Pd	7238	74.6	-0.083	-0.181	0.510	0
$^{120}_{46}$Pd	5371.5	294	-0.709	0.415	0.518	0
$^{121}_{46}$Pd	8220	1397.9	-1.238	-0.653	0.510	$0.10^{-0.06}_{+0.15}$
$^{122}_{46}$Pd	6490	1715	-1.645	-0.321	0.518	$0.55^{-0.31}_{+0.76}$
$^{128}_{46}$Pd	10130	5880	-3.352	-1.929	0.523	$25.28^{-12.99}_{+19.80}$
$^{129}_{46}$Pd	14370	8940	-3.474	-2.738	0.513	$39.52^{-18.22}_{+21.72}$
$^{130}_{47}$Ag	15420	9290	-3.194	-2.151	0.519	$48.29^{-20.51}_{+21.11}$
$^{131}_{47}$Ag	14840	12670	-3.352	-2.946	0.513	$71.84^{-20.58}_{+14.20}$
$^{132}_{47}$Ag	16470	13360	-3.576	-2.590	0.517	$75.48^{-19.73}_{+12.71}$
$^{126}_{48}$Cd	5516	149	-0.666	0.980	0.521	0
$^{127}_{48}$Cd	8149	954	-0.799	0.036	0.513	$0.04^{-0.02}_{+0.06}$
$^{128}_{48}$Cd	6900	1583	-1.402	-0.065	0.522	$0.54^{-0.31}_{+0.76}$
$^{129}_{48}$Cd	9780	3020	-1.890	-0.825	0.514	$2.94^{-1.68}_{+3.87}$

Continued on next column

Nucl.	Q_β	$Q_{\beta n}$	$\ln T_{exp}$	$\ln T_{pred}$	$\sigma_{\ln T_{pred}}$	$P_{cal}(\%)$
$^{133}_{48}$Cd	13540	10420	-2.749	-2.502	0.513	$61.77^{-21.77}_{+17.86}$
$^{134}_{48}$Cd	12740	10470	-2.733	-3.206	0.521	$58.49^{-22.00}_{+19.03}$
$^{135}_{49}$In	14100	11830	-2.293	-2.689	0.513	$70.92^{-20.80}_{+14.58}$
$^{136}_{49}$In	15390	12050	-2.465	-2.197	0.518	$74.18^{-20.15}_{+13.28}$
$^{137}_{49}$In	14750	12790	-2.733	-2.946	0.513	$73.19^{-20.24}_{+13.64}$
$^{138}_{50}$Sn	9400	7130	-1.871	-1.616	0.523	$52.33^{-21.38}_{+20.55}$
$^{139}_{50}$Sn	11350	7700	-2.120	-1.593	0.514	$49.58^{-20.70}_{+20.85}$
$^{134}_{51}$Sb	8513.2	845.3	-0.393	0.772	0.519	$0.03^{-0.02}_{+0.05}$
$^{141}_{51}$Sb	11380	9400	-2.273	-1.721	0.513	$67.87^{-21.30}_{+15.75}$
$^{139}_{52}$Te	8265.9	3703.5	-0.298	-0.189	0.512	$11.86^{-6.52}_{+12.56}$
$^{140}_{52}$Te	7030	3823	-1.047	-0.404	0.520	$16.94^{-9.10}_{+16.05}$
$^{141}_{52}$Te	9440	5050	-1.645	-0.978	0.511	$20.57^{-10.80}_{+17.76}$
$^{142}_{52}$Te	8400	5490	-1.917	-1.428	0.519	$28.61^{-14.39}_{+20.69}$
$^{143}_{52}$Te	10350	6420	-2.120	-1.577	0.510	$30.16^{-14.89}_{+20.74}$
$^{142}_{53}$I	10460	5360	-1.448	-0.806	0.514	$21.38^{-11.18}_{+18.14}$
$^{143}_{53}$I	9570	6530	-1.704	-1.403	0.510	$34.84^{-16.61}_{+21.37}$
$^{144}_{53}$I	11590	6850	-2.364	-1.506	0.513	$29.69^{-14.77}_{+20.81}$
$^{145}_{53}$I	10550	7860	-2.411	-2.056	0.510	$39.97^{-18.28}_{+21.60}$
$^{148}_{54}$Xe	8310	5250	-2.465	-2.060	0.518	$15.04^{-8.17}_{+15.03}$
$^{151}_{55}$Cs	10710	7600	-2.830	-2.403	0.511	$29.52^{-14.67}_{+20.70}$
$^{150}_{56}$Ba	6230	2250	-1.355	-0.674	0.518	$1.43^{-0.82}_{+1.94}$
$^{151}_{56}$Ba	8370	3120	-1.790	-1.185	0.511	$2.36^{-1.35}_{+3.12}$
$^{152}_{56}$Ba	7580	3530	-1.966	-1.655	0.518	$4.27^{-2.43}_{+5.44}$
$^{153}_{56}$Ba	9590	4750	-2.180	-1.866	0.511	$7.85^{-4.39}_{+9.10}$
$^{154}_{56}$Ba	8710	5170	-2.937	-2.350	0.518	$11.57^{-6.39}_{+12.51}$
$^{151}_{57}$La	7910	3470	-0.783	-0.917	0.511	$4.74^{-2.69}_{+5.91}$
$^{152}_{57}$La	9690	3860	-1.211	-0.988	0.513	$5.03^{-2.85}_{+6.25}$
$^{153}_{57}$La	8850	4850	-1.406	-1.478	0.511	$11.76^{-6.47}_{+12.45}$
$^{154}_{57}$La	10690	5310	-1.826	-1.479	0.513	$12.37^{-6.79}_{+13.00}$
$^{155}_{57}$La	9850	6220	-2.293	-2.014	0.511	$19.98^{-10.53}_{+17.51}$
$^{156}_{57}$La	11770	6660	-2.477	-1.961	0.513	$20.1^{-10.62}_{+17.73}$

Continued on next column

Nucl.	Q_β	$Q_{\beta n}$	$\ln T_{exp}$	$\ln T_{pred}$	$\sigma_{\ln T_{pred}}$	$P_{cal}(\%)$
$^{85}_{30}$Zn	14620	10790	-	-2.682	0.512	$54.96^{-21.41}_{+19.71}$
$^{89}_{32}$Ge	13070	8920	-	-2.005	0.514	$50.23^{-20.82}_{+20.73}$
$^{90}_{32}$Ge	12110	9510	-	-2.589	0.523	$56.10^{-21.84}_{+19.68}$
$^{89}_{33}$As	12190	9020	-	-1.732	0.513	$57.61^{-21.66}_{+19.07}$
$^{90}_{33}$As	14470	9590	-	-1.666	0.518	$57.12^{-21.75}_{+19.28}$
$^{91}_{33}$As	13680	10830	-	-2.351	0.513	$63.67^{-21.69}_{+17.23}$
$^{92}_{33}$As	15740	11530	-	-2.134	0.517	$66.32^{-21.65}_{+16.39}$
$^{92}_{34}$Se	9510	6310	-	-1.761	0.519	$29.99^{-14.94}_{+20.99}$
$^{93}_{34}$Se	12180	7450	-	-2.103	0.510	$28.87^{-14.38}_{+20.47}$
$^{94}_{34}$Se	10600	8020	-	-2.444	0.518	$39.83^{-18.46}_{+21.94}$
$^{95}_{34}$Se	13310	8870	-	-2.685	0.510	$33.98^{-16.31}_{+21.29}$
$^{95}_{35}$Br	12390	9510	-	-2.575	0.510	$43.17^{-19.18}_{+21.48}$
$^{96}_{35}$Br	14920	9920	-	-2.624	0.513	$38.11^{-17.87}_{+21.84}$
$^{97}_{35}$Br	13370	10950	-	-3.070	0.510	$47.37^{-20.20}_{+21.11}$
$^{98}_{35}$Br	16060	11100	-	-3.082	0.513	$40.00^{-18.50}_{+21.91}$
$^{101}_{36}$Kr	13720	9050	-	-3.342	0.511	$23.31^{-12.05}_{+18.91}$
$^{107}_{38}$Sr	13470	9080	-	-3.287	0.511	$25.01^{-12.80}_{+19.50}$
$^{152}_{55}$Cs	12780	7940	-	-2.344	0.513	$27.55^{-13.96}_{+20.47}$

IV. CONCLUSION

This work focuses on the properties of β-decay: the half-lives and the probability of releasing delayed neutrons of neutron-rich nuclei with the atomic number from 29 to 57, which are important fission products. In considering the odevity as well as the shell effect, phenomenological formulas for β-decay are proposed on top of the classical formula. For the β-decay neutron emission (βn) probability, it has a similar formula as the half-life based on their relationship analysis, except for new terms to include the differences between the decay energy of releasing delayed neutrons and that of not.

Based on the fitting results, the β-decay half-lives, βn probabilities, and the corresponding uncertainties have been calculated. The experimental half-lives are generally well reproduced, in particular, the shorter the half-life, the better the consistency. The uncertainty analysis of β-decay formula is successfully performed by the bootstrap method. In this way, the uncertainties of the theoretically predicted values are given, which helps to better understand the disparity between experimental and theoretical results and to predict β-decay half-lives and βn probabilities for nuclei without experimental data.

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[1] R. B. Roberts, R. C. Meyer, P. Wang, Further Observations on the Splitting of Uranium and Thorium. Phys. Rev. **55**, 0510-0511 (1939). doi:10.1103/PhysRev.55.510.2

[2] R. Barton, R. Mcpherson, and R. E. Bell, *et al.*, Observation of delayed proton radioactivity. Can. J. Phys. **41**, 2007-2025 (1963). doi:10.1139/p63-201

[3] M. D. Cable, J. Honkanen, R. F. Parry, *et al.*, Discovery of beta-delayed two-proton radioactivity: ^{22}Al. Phys. Rev. Lett. **50**, 404-406 (1983). doi:10.1103/PhysRevLett.50.404

[4] S. R. Elliott, A. A. Hahn, M. K. Moe, Direct evidence for two-neutrino double-β decay in ^{82}Se. Phys. Rev. Lett. (1987). doi:10.1103/PhysRevLett.59.2020

[5] E. Gete, L. Buchmann, R. E. Azuma, *et al.*, β-delayed particle decay of ^{9}C and the $A = 9$, $T = 1/2$ nuclear system: Experiment, data, and phenomenological analysis. Phys. Rev. C **61**, 064310 (2000). doi:10.1103/PhysRevC.61.064310

[6] K. Miernik, W. Dominik, Z. Janas, *et al.*, First observation of β-delayed three-proton emission in ^{45}Fe. Phys. Rev. C **76**, 041304 (2007). doi:10.1103/PhysRevC.76.041304

[7] M. J. G. Borge, β-delayed particle emission. Phys. Scr. **T152**, 014013 (2013). doi:10.1088/0031-8949/2013/t152/014013

[8] S. E. A. Orrigo, B. Rubio, Y. Fujita, *et al.*, Observation of the β-delayed γ-proton decay of ^{56}Zn and its impact on the Gamow-Teller strength evaluation. Phys. Rev. Lett. **112**, 222501 (2014). doi:10.1103/PhysRevLett.112.222501

N. Frédéric, O. Alexandre, P. Alfredo, The neutron-rich edge of the nuclear landscape: Experiment and theory. Prog. Part. Nucl. Phys. **120**, 103866 (2021). doi:https://doi.org/10.1016/j.ppnp.2021.103866

L. Zhou, S. M. Wang, D. Q. Fang, *et al.*, Recent progress in two-proton radioactivity. Nucl. Sci. Tech. **33**, 105 (2022). doi:10.1007/s41365-022-01091-1

X. P. Zhang, Z. Z. Ren, Q. J. Zhi, *et al.*, Systematics of β^--decay half-lives of nuclei far from the β-stable line. J. Phys. G Nucl. Part. Phys. **34**, 2611-2632 (2007). doi:10.1088/0954-3899/34/12/007

B. Pfeiffer, K. L. Kratz, P. Möller, *et al.*, Status of delayed-neutron precursor data: half-lives and neutron emission probabilities. Prog. Nucl. Energy **41**, 39-69 (2002). doi:https://doi.org/10.1016/S0149-1970(02)00005-7

H. M. Şahin, G. Tunç, A. Karakoç, *et al.*, Neutronic study on the effect of first wall material thickness on tritium production and material damage in a fusion reactor. Nucl. Sci. Tech. **33**, 39-69 (2022). doi:10.1007/s41365-022-01029-7

Y. Zhou, Z. H. Li, Y. B. Wang, *et al.*, Empirical formula for β^--decay half-lives of r-process nuclei. Sci. China. Phys. Mech. Astron. **60**, 082012 (2017). doi:10.1007/s11433-017-9045-0

[15] Z. M. Niu., Y. F. Niu, H. Z. Liang, *et al.*, β-decay half-lives of neutron-rich nuclei and matter flow in the r-process. Phys. Lett. B **723**, 172-176 (2013). doi:https://doi.org/10.1016/j.physletb.2013.04.048

[16] P. Möller, M. R. Mumpower, T. Kawano, Nuclear properties for astrophysical and radioactive-ion-beam applications (II). Atom. Data Nucl. Data Tables **125**, 1-192 (2019). doi:https://doi.org/10.1016/j.adt.2018.03.003

[17] S. Yoshida, Y. Utsuno, N. Shimizu, *et al.*, Systematic shell-model study of β-decay properties and Gamow-Teller strength distributions in $A \approx 40$ neutron-rich nuclei. Phys. Rev. C **97**, 054321 (2018). doi:10.1103/PhysRevC.97.054321

[18] I. N. Borzov, S. Goriely, J. M. Pearson, Microscopic calculations of β-decay characteristics near the $A = 130$ r-process peak. Nucl. Phys. A **81**, 044620 (2010). doi: 10.1103/PhysRevC.81.044620

[19] I. N. Borzov, J. J. Cuenca-García, K. Langanke, *et al.*, Beta-decay of $Z < 50$ nuclei near the $N = 82$ closed neutron shell. Nucl. Phys. A **814**, 159-173 (2008). doi: 10.1016/j.nuclphysa.2008.09.010

[20] J. J. Cuenca-García, G. Martínez-Pinedo, K. Langanke, *et al.*, Shell model half-lives for r-process $N = 82$ nuclei. Eur. Phys. J. A **34**, 99-105 (2007). doi:10.1140/epja/i2007-10477-3

[21] K. Yoshida, Suddenly shortened half-lives beyond ^{78}Ni : $N = 50$ magic number and high-energy nonunique first-forbidden. Phys. Rev. C **100**, 024316 (2019). doi:10.1103/PhysRevC.100.024316

[22] X. P. Zhang, Z. Z. Ren, New exponential law of β^+-decay half-lives of nuclei far from β-stable line. Phys. Rev. C **73**, 014305 (2006). doi:10.1103/PhysRevC.73.014305

[23] K. Miernik, Phenomenological model of β-delayed neutron-emission probability. Phys. Rev. C **88**, 041301 (2013). doi:10.1103/PhysRevC.88.041301

[24] A. I. Morales, J. Benlliure, T. Kurtukián-Nieto, *et al.*, Half-Life Systematics across the $N = 126$ Shell closure: Role of first-forbidden transitions in the β decay of heavy neutron-rich nuclei. Phys. Rev. Lett. **113** 022702 (2014). doi:10.1103/PhysRevLett.113.022702

[25] R. J. Carroll, Zs. Podolyák, T. Berry, *et al.*, Competition between allowed and first-forbidden β decay: The case of ^{208}Hg $\rightarrow ^{208}$Tl. Phys. Rev. Lett. **125**, 192501 (2020). doi:10.1103/PhysRevLett.125.192501s

[26] G. Audi, A. H. Wapstra, C. Thibault, The AME2003 atomic mass evaluation: (II). Tables, graphs and references. Nucl. Phys. A **729**, 337-676 (2003). doi:https://doi.org/10.1016/j.nuclphysa.2003.11.003

[27] J. Pereira, S. Hennrich, A. Aprahamian, *et al.*, β-decay half-lives and β-delayed neutron emission probabilities of nuclei in the region $A \lesssim 110$, relevant for the r process. Phys. Rev. C **79**, 035806 (2009). doi:10.1103/PhysRevC.79.035806

[28] P. T. Hosmer, H. Schatz, A. Aprahamian, *et al.*, Half-Life of the doubly magic r-process nucleus ^{78}Ni. Phys. Rev. Lett. **94**, 112501. doi:10.1103/PhysRevLett.94.112501

[29] R. Dunlop, V. Bildstein, I. Dillmann, *et al.*, Half-lives of neutron-rich $^{128-130}$Cd. Phys. Rev. C **93**, 062801 (2016). doi:10.1103/PhysRevC.93.062801

[30] T. Marketin, L. Huther, G. Martínez-Pinedo, Large-scale evaluation of β-decay rates of r-process nuclei with the inclusion of first-forbidden transitions. Phys. Rev. C **93**, 025805 (2016). doi:10.1103/PhysRevC.93.025805

[31] J. L. You, X. P. Zhang, Q. J. Zhi, *et al.*, The contribution of the first forbidden transitions to the nuclear β^--decay half-life. Chinese Phys. C **43**, 114104 (2019). doi:10.1088/1674-1137/43/11/114104

[32] C. L. Lan, M. Peng, Y .Zhang, *et al.*, Possible cluster states in heavy and superheavy nuclei. Nucl. Sci. Tech. **104**, L031301 (2017). doi:10.1007/s41365-016-0158-7

[33] M. Wang, W. J. Huang, F. G. Kondev, *et al.*, The AME 2020 atomic mass evaluation (II). Tables, graphs and references∗, Chinese Phys. C **45**, 030003 (2021). doi:10.1088/1674-1137/abddaf

[34] J. Liang, B. Singh, E. A. McCutchan, I. Dillmann, *et al.*, Compilation and evaluation of β-delayed neutron emission probabilities and half-Lives for $Z > 28$ precursors. Nucl. Data Sheets **168**, 1-116 (2020). 10.1016/j.nds.2020.09.001

[35] J. Suhonen, From nucleons to nucleus: Concepts of microscopic nuclear theory (Springer Science & Business Media, 2007), pp. 163-164.

[36] H. Primakoff, S. P. Rosen, Double beta decay. Rep. Prog. Phys. **22**, 121 (1959). doi:10.1088/0034-4885/22/1/305

[37] B. Efron, R. Tibshirani, The bootstrap method for assessing statistical accuracy. Behaviormetrika **12**, 1-35 (1985). doi:10.2333/bhmk.12.17_1

[38] B. S. Cai, G. S. Chen, J. Y. Xu, *et al.*, α decay half-life estimation and uncertainty analysis. Phys. Rev. C **101**, 054304 (2020). doi:10.1103/PhysRevC.101.054304

[39] Y. P. Cao, D. H. Lu, Y. B. Qian, *et al.*, Uncertainty analysis for the nuclear liquid drop model and implications for the symmetry energy coefficients. Phys. Rev. C **105**, 034304 (2022). doi:10.1103/PhysRevC.105.034304

[40] B. S. Cai, G. S. Chen, C. X. Yuan, *et al.*, Shell-model study on properties of proton dripline nuclides with Z, N = 30 − 50 including uncertainty analysis. Chinese Phys. C (2022). doi:10.1088/1674-1137/ac6cd7,

[41] J. H. Jia, Y. B. Qian, Z. Z. Ren, Possible cluster states in heavy and superheavy nuclei. Phys. Rev. C **104**, L031301 (2021). doi:10.1103/PhysRevC.104.L031301

[42] Z. X. Fang, M. Yu, Y. G. Huang, *et al.*, Theoretical analysis of long-lived radioactive waste in pressurized water reactor. Nucl. Sci. Tech. **32**, 72 (2021). doi:10.1007/s41365-021-00911-0

[43] G. Martínez-Pinedo, K. Langanke, Shell model applications in nuclear astrophysics. Physics **4**, 677-689 (2022). doi:10.3390/physics4020046